Suggested Solutions to:
Regular Exam, Fall 2018
Contract Theory
January 10, 2019

This version: January 28, 2019

Question 1: Information gathering in the adverse selection model

Part (a)

- The constraints IG-bad and IG-good are the ones that ensure that the firm has an incentive to gather information.
  - The left-hand side of each of these constraints is the firm's expected payoff at the stage of information acquisition, given that it indeed acquires information and then chooses the contract that the procurement agency wants it to choose (this is the best the firm can do if having acquired information, given that the IR and IC constraints are not violated).
  - The right-hand side of IG-bad is the expected payoff the firm would get if not acquiring information (thus not learning its type) and then picking the contract meant for the bad type; the right-hand side of IG-good is the same, but for the case where the firm picks the contract meant for the good type. If the firm did not acquire information, then it would pick either the bad or the good type's contract, depending on which one gave the highest expected payoff (at least one of these options must be better than the outside option payoff of zero). Therefore both IG-bad and IG-good must hold to ensure that the firm wants to acquire information.
- The constraint IR-ante is a participation constraint at the ex ante stage when the agent has not yet acquired information. It ensures that the agent, at the ex ante stage when she does not know her type, does not prefer her outside option.

Part (b)

- Before solving the problem, it is useful to try to simplify it by eliminating some of the constraints.

Lemma 1 *IG-good implies IC-bad.*

Proof. We can rewrite IG-good as

$$(1-\nu)\left[\bar{t}-\underline{t}+C\left(\underline{q},\bar{\theta}\right)-C\left(\bar{q},\bar{\theta}\right)\right]\geq\gamma.$$

Both $(1-\nu)$ and γ are strictly positive. Therefore the expression in square brackets must be non-negative for the inequality to hold. But the expression in square brackets being non-negative is exactly the IC-bad constraint. This proves that IG-good implies IC-bad. □

Lemma 2 *IG-bad implies IC-good.*

Proof. We can rewrite IG-bad as

$$\nu\left[\underline{t}-\bar{t}+C\left(\bar{q},\underline{\theta}\right)-C\left(\underline{q},\underline{\theta}\right)\right]\geq\gamma.$$

Both ν and γ are strictly positive. Therefore the expression in square brackets must be non-negative for the inequality to hold. But the expression in square brackets being non-negative is exactly the IC-good constraint. This proves that IG-bad implies IC-good. □

Lemma 3 *IC-good and IR-bad jointly imply IR-good.*

Proof. We can write:

$$\underline{t}-C\left(\underline{q},\underline{\theta}\right)\overset{\text{(i)}}{\geq}\bar{t}-C\left(\bar{q},\underline{\theta}\right)\overset{\text{(ii)}}{\geq}\bar{t}-C\left(\bar{q},\bar{\theta}\right)\overset{\text{(iii)}}{\geq}0.$$

Here inequality (i) follows from IC-good. Inequality (ii) follows from the fact that the cost function is increasing in θ and $\bar{\theta}>\underline{\theta}$ (i.e., a bad type produces a given quantity at a higher cost). Inequality (iii) follows from IR-bad. The above series of inequalities says in particular that $\underline{t}-C\left(\underline{q},\underline{\theta}\right)\geq 0$, which is IR-good. Hence we have proven that IC-good and IR-bad jointly imply IR-good. □

Lemma 4 *IG-bad and IR-bad jointly imply IR-ante.*

Proof. IG-bad can be written as

$$\begin{aligned}\nu\left[\underline{t}-C\left(\underline{q},\underline{\theta}\right)\right]+(1-\nu)\left[\bar{t}-C\left(\bar{q},\bar{\theta}\right)\right]-\gamma &\geq \bar{t}-\nu C\left(\bar{q},\underline{\theta}\right)-(1-\nu)\,C\left(\bar{q},\bar{\theta}\right)\\ &= \underbrace{\bar{t}-C\left(\bar{q},\bar{\theta}\right)}_{\geq 0\text{ by IR-bad}}+\nu\left[\underbrace{C\left(\bar{q},\bar{\theta}\right)-C\left(\bar{q},\underline{\theta}\right)}_{>0\text{ by }C_\theta>0}\right]\\ &> 0,\end{aligned}$$

where (as indicated) the last inequality makes use of IR-bad and of the assumptions that $C_\theta\left(\bar{q},\theta\right)>0$ and $\bar{\theta}>\underline{\theta}$. □

- The above results mean that we safely can ignore IC-bad, IC-good, IR-ante and IR-good, as they are implied by the other constraints.
- The Lagrangian associated with the remaining problem is:

$$\begin{aligned}\mathcal{L} = {} & \nu\left[S\left(\underline{q}\right)-\underline{t}\right]+(1-\nu)\left[S\left(\bar{q}\right)-\bar{t}\right]+\lambda\left[\bar{t}-C\left(\bar{q},\bar{\theta}\right)\right]\\ &+\underline{\mu}\left[(1-\nu)\left[\bar{t}-\underline{t}+C\left(\underline{q},\bar{\theta}\right)-C\left(\bar{q},\bar{\theta}\right)\right]-\gamma\right]\\ &+\bar{\mu}\left[\nu\left[\underline{t}-\bar{t}+C\left(\bar{q},\underline{\theta}\right)-C\left(\underline{q},\underline{\theta}\right)\right]-\gamma\right],\end{aligned}$$

where λ (≥ 0) is the shadow price of IR-bad, $\underline{\mu}$ (≥ 0) is the shadow price of IG-good, and $\overline{\mu}$ (≥ 0) is the shadow price of IG-bad.

- Differentiating the Lagrangian with respect to the choice variables $\underline{t}$, $\overline{t}$, $\underline{q}$, and $\overline{q}$, and then setting the resulting expression equal to zero, yields the following first-order conditions.

- FOC w.r.t. $\underline{t}$:

$$\frac{\partial \mathcal{L}}{\partial \underline{t}} = -\nu - \underline{\mu}(1-\nu) + \overline{\mu}\nu = 0. \tag{1}$$

- FOC w.r.t. $\overline{t}$:

$$\frac{\partial \mathcal{L}}{\partial \overline{t}} = -(1-\nu) + \lambda + \underline{\mu}(1-\nu) - \overline{\mu}\nu = 0. \tag{2}$$

- FOC w.r.t. $\underline{q}$:

$$\frac{\partial \mathcal{L}}{\partial \underline{q}} = \nu S'\left(\underline{q}\right) + \underline{\mu}(1-\nu) C_q\left(\underline{q},\overline{\theta}\right) - \overline{\mu}\nu C_q\left(\underline{q},\underline{\theta}\right) = 0. \tag{3}$$

- FOC w.r.t. $\overline{q}$:

$$\frac{\partial \mathcal{L}}{\partial \overline{q}} = (1-\nu) S'(\overline{q}) - \lambda C_q(\overline{q},\overline{\theta}) - \underline{\mu}(1-\nu) C_q(\overline{q},\overline{\theta}) + \overline{\mu}\nu C_q(\overline{q},\underline{\theta}) = 0. \tag{4}$$

- Adding (1) and (2) yields

$$\lambda = 1. \tag{5}$$

The fact that $\lambda > 0$ implies that IR-bad binds.

- We can rewrite (1) as

$$\overline{\mu}\nu = \underline{\mu}(1-\nu) + \nu. \tag{6}$$

Since $\nu \in (0,1)$ and $\underline{\mu} \geq 0$, this implies that $\overline{\mu} > 0$; hence, IG-bad binds.

- Rewrite (4) using (5) and (6):

$$\begin{aligned}
(1-\nu) S'(\overline{q}) &= \lambda C_q(\overline{q},\overline{\theta}) + \underline{\mu}(1-\nu) C_q(\overline{q},\overline{\theta}) - \overline{\mu}\nu C_q(\overline{q},\underline{\theta}) \\
&= C_q(\overline{q},\overline{\theta}) + \underline{\mu}(1-\nu)\left[C_q(\overline{q},\overline{\theta}) - C_q(\overline{q},\underline{\theta})\right] - \nu C_q(\overline{q},\underline{\theta}) \\
&= (1-\nu) C_q(\overline{q},\overline{\theta}) + \underline{\mu}(1-\nu)\left[C_q(\overline{q},\overline{\theta}) - C_q(\overline{q},\underline{\theta})\right] \\
&\quad +\nu\left[C_q(\overline{q},\overline{\theta}) - C_q(\overline{q},\underline{\theta})\right]
\end{aligned}$$

$$\Leftrightarrow S'(\overline{q}) = C_q(\overline{q},\overline{\theta}) + \underbrace{\left[\underline{\mu} + \frac{\nu}{1-\nu}\right]}_{=\frac{\nu\overline{\mu}}{1-\nu},\text{ where }\overline{\mu}>0} \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} C_{q\theta}(\overline{q},\theta)\, d\theta}_{>0}.$$

$$\Rightarrow \overline{q}^{SB} < \overline{q}^{FB}.$$

- That is, the bad type's second-best quantity is strictly lower than the first-best quantity, which is one of the things that was asked about in the question.

- Now rewrite (3) using (6):

$$\begin{aligned}
\nu S'\left(\underline{q}\right) &= \overline{\mu}\nu C_q\left(\underline{q},\underline{\theta}\right) - \underline{\mu}(1-\nu) C_q\left(\underline{q},\overline{\theta}\right) \\
&= \nu C_q\left(\underline{q},\underline{\theta}\right) - \underline{\mu}(1-\nu)\left[C_q\left(\underline{q},\overline{\theta}\right) - C_q\left(\underline{q},\underline{\theta}\right)\right] \Leftrightarrow
\end{aligned}$$

$$S'\left(\underline{q}\right) = C_q\left(\underline{q}, \underline{\theta}\right) - \frac{\underline{\mu}(1-\nu)}{\nu} \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} C_{q\theta}\left(\underline{q}, \theta\right) d\theta}_{>0}.$$

$$\Rightarrow \underline{q}^{SB} \geq \underline{q}^{FB} \qquad \text{(with equality if and only if } \underline{\mu} = 0\text{)}.$$

- That is, the good type's second-best quantity is either the same as the first-best quantity or larger, which was the second thing asked about in the question.

Part (c)

The results derived above do not answer the question whether the good type's quantity is distorted upwards or if it equals the first-best quantity. However, it is possible to show that we do indeed, also in this model, have efficiency at the top ($\underline{q}^{SB} = \underline{q}^{FB}$), provided that it is optimal for P to induce an effort from the agent. In particular, one can prove the following result:

Lemma 5 *Suppose it is optimal for P to induce information gathering. Then, at the optimum, IG-good and IG-bad cannot both bind.*

Together with the results in the (b) part, Lemma 5 implies that IG-good must be lax (because IG-bad binds). This in turn means that there is efficiency at the top ($\underline{q}^{SB} = \underline{q}^{FB}$).

In the course we showed Lemma 5 with the help of a proof by contradiction. We thus supposed that P optimally induces information gathering ($x = 1$), chooses an optimal menu of contracts, and that IG-good and IG-bad both bind. We then showed that, under these circumstances, P can earn higher profits by not inducing information gathering ($x = 0$) and, instead of the supposedly optimal menu, offer a pooled (single) contract. In this pooled contract, the single quantity is a convex combination of the quantities in the original contract (where the weight on each type's quantity is given by the type's prior probability). Similarly, the transfer in the pooled contract is a convex combination of the transfers in the original contract (again, with weights given by type probability).

To complete the proof, we had to do two things:

1. We had to show that the pooled contract indeed gave rise to a higher expected profit for P. We could do this by first writing up the profit expression evaluated at the original contract. By rewriting this expression, using Jensen's inequality and the strict concavity of the $S(q)$ function, we could show that the expression was strictly smaller than the same expression evaluated at the pooled contract (it here helped us that the latter contract was defined as a convex combination of the former).

2. We also had to show that all the constraints were satisfied at the pooled contract. By inspecting these constraints, it was clear that the only one that mattered now when the menu consisted of a single contract was IR-ante. To show that IR-ante was satisfied at the pooled contract, we made use of the fact both IG constraints were binding at the original contract. We thus changed the inequalities to equalities in IG-good and IG-bad and evaluated them at the original contract. We then considered a convex combination of each side of the two equalities, with the weights (again) given by the type probabilities. This obviously did not change the left-hand side of the expressions, as they were identical to start with. The left-hand side of the new equality was therefore equal to

the left-hand side of IR-ante, evaluated at the original contract. The right-hand side of the new equality could —thanks to Jensen's inequality and the convexity of the cost function—be rewritten as a weakly larger expression (it here again helped us that the pooled contract was defined as a convex combination of the original contract). The right-hand side of the new equality also turned out to equal the left-hand side of IR-ante, evaluated at the pooled contract. It thus followed that the IR-ante constraint is easier to satisfy at the pooled contract than at the original contract. Since it was indeed satisfied at the original contract, it must therefore also hold at the pooled contract.

Further information to the external examiner

Below follows the formal proof of Lemma 5. This is of course not required in a student's answer, but I make it available to you in case it might help you when grading.

Proof of Lemma 5. Suppose, per contra, that P optimally induces information gathering ($x = 1$), chooses an optimal menu of contracts, and that IG-good and IG-bad both bind. Below it will be shown that, in this situation, P can earn higher profits by not inducing information gathering ($x = 0$) and, instead of the supposedly optimal menu, offer a pooled contract

$$(q^p, t^p) = \left(\nu \underline{q}^{SB} + (1-\nu)\,\overline{q}^{SB}, \nu \underline{t}^{SB} + (1-\nu)\,\overline{t}^{SB}\right).$$

Moreover, this pooled contract is feasible and incentive compatible. Hence we have a contradiction, which means that IG-good and IG-bad cannot both bind. P's payoff if choosing $x = 1$ and offering the supposedly optimal menu can be written as

$$\nu \left[S\left(\underline{q}^{SB}\right) - \underline{t}^{SB}\right] + (1-\nu)\left[S\left(\overline{q}^{SB}\right) - \overline{t}^{SB}\right] < S\left[\nu \underline{q}^{SB} + (1-\nu)\,\overline{q}^{SB}\right] - \nu \underline{t}^{SB} - (1-\nu)\,\overline{t}^{SB} = S\left(q^p\right) - t^p,$$

where the inequality follows from $S'' < 0$ and Jensen's inequality. Thus, the principal's payoff is strictly larger if offering the pooled contract. If the principal does not induce information gathering, there is only one constraint in the profit maximization problem, namely a (modified) IR-ante constraint that reads

$$t^p - \nu C\left(q^p, \underline{\theta}\right) - (1-\nu)\,C\left(q^p, \overline{\theta}\right) \geq 0. \tag{7}$$

We must verify that this holds. By assumption, IG-good and IG-bad bind, so we have

$$\nu \left[\underline{t} - C\left(\underline{q}^{SB}, \underline{\theta}\right)\right] + (1-\nu)\left[\overline{t} - C\left(\overline{q}^{SB}, \overline{\theta}\right)\right] - \gamma = \underline{t} - \nu C\left(\underline{q}^{SB}, \underline{\theta}\right) - (1-\nu)\,C\left(\underline{q}^{SB}, \overline{\theta}\right), \tag{8}$$

$$\nu \left[\underline{t} - C\left(\underline{q}^{SB}, \underline{\theta}\right)\right] + (1-\nu)\left[\overline{t} - C\left(\overline{q}^{SB}, \overline{\theta}\right)\right] - \gamma = \overline{t} - \nu C\left(\overline{q}^{SB}, \underline{\theta}\right) - (1-\nu)\,C\left(\overline{q}^{SB}, \overline{\theta}\right). \tag{9}$$

Now multiply both sides of (8) by ν, and multiply both sides of (9) by $1-\nu$. Then add up the resulting expressions, to obtain:

$$\begin{aligned}
& \nu \left[\underline{t} - C\left(\underline{q}^{SB}, \underline{\theta}\right)\right] + (1-\nu)\left[\overline{t} - C\left(\overline{q}^{SB}, \overline{\theta}\right)\right] - \gamma \\
= \;& \nu \left[\underline{t} - \nu C\left(\underline{q}^{SB}, \underline{\theta}\right) - (1-\nu)\,C\left(\underline{q}^{SB}, \overline{\theta}\right)\right] + (1-\nu)\left[\overline{t} - \nu C\left(\overline{q}^{SB}, \underline{\theta}\right) - (1-\nu)\,C\left(\overline{q}^{SB}, \overline{\theta}\right)\right] \\
= \;& t^p - \nu \left[\nu C\left(\underline{q}^{SB}, \underline{\theta}\right) + (1-\nu)\,C\left(\overline{q}^{SB}, \underline{\theta}\right)\right] - (1-\nu)\left[\nu C\left(\underline{q}^{SB}, \overline{\theta}\right) + (1-\nu)\,C\left(\overline{q}^{SB}, \overline{\theta}\right)\right] \\
\leq \;& t^p - \nu C\left[\nu \underline{q}^{SB} + 1 - \nu \overline{q}^{SB}, \underline{\theta}\right] - (1-\nu)\,C\left[\nu \underline{q}^{SB} + 1 - \nu \overline{q}^{SB}, \overline{\theta}\right] \\
= \;& t^p - \nu C\left(q^p, \underline{\theta}\right) - (1-\nu)\,C\left(q^p, \overline{\theta}\right),
\end{aligned}$$

where the inequality follows from $C_{qq}(q, \theta) \geq 0$ and Jensen's inequality. Moreover, we know that since the IR-ante constraint in the original problem holds, the left-hand side of (6) is non-negative. It follows that the modified IR-ante constraint in (7) is indeed satisfied. $\square$

Question 2: Sharecropping with a continuum of effort and output levels

Part (a)

(a) Characterize the second-best optimal value of α, using the first-order approach.[1] Assume that the second-order conditions are satisfied (you will not get any credit if you nevertheless investigate whether these conditions hold).

The landlord's problem is to choose $\alpha \in [0,1]$ and $e \in [0,\infty)$ so as to maximize the expected profits $EV = \frac{(1-\alpha)e}{1+e}$, subject to the following constraints:

$$EU = \frac{\alpha e}{1+e} - ce \geq 0, \tag{IR}$$

$$e \in \arg\max_{e'} \frac{\alpha e'}{1+e'} - ce'. \tag{IC}$$

We can first note that the IR constraint is implied by the IC constraint. (The payoff zero can be achieved by setting $e = 0$; therefore, any e that satisfies IC yields a payoff of at least zero, which means that it also satisfies IR.) We can thus ignore the IR constraint.

We solve the remaining problem by using the first-order approach, which here means that we first solve for the farmer's optimal effort level for any given contract parameter α; let us denote this optimal effort level by $e^*(\alpha)$. Then we let $e^*(\alpha)$ represent the IC constraint above (which really is a whole set of constraints). The first-order condition associated with the agent's effort choice problem is given by

$$\frac{\partial EU}{\partial e} = \frac{\alpha}{(1+e)^2} - c \leq 0.$$

Solving for e, while taking the non-negativity constraint into account, we have

$$e^*(\alpha) = \max\left\{0, \sqrt{\frac{\alpha}{c}} - 1\right\}.$$

So if $\alpha \leq c$, then $e^*(\alpha) = 0$; otherwise, $e^*(\alpha) = \sqrt{\frac{\alpha}{c}} - 1$.

Clearly, if the landlord sets $\alpha \leq c$, her profit is zero. Suppose $\alpha > c$. Then, if we plug in $e^*(\alpha)$, the landlord's objective simplifies to

$$EV = \frac{(1-\alpha)\, e^*(\alpha)}{1 + e^*(\alpha)} = \frac{(1-\alpha)\left(\sqrt{\frac{\alpha}{c}} - 1\right)}{\sqrt{\frac{\alpha}{c}}} = \frac{(1-\alpha)\left(\sqrt{\alpha} - \sqrt{c}\right)}{\sqrt{\alpha}} = (1-\alpha)\left(1 - c^{\frac{1}{2}}\alpha^{-\frac{1}{2}}\right).$$

Taking the first-order condition with respect to the choice variable α yields

$$\frac{\partial EV}{\partial \alpha} = -\left(1 - c^{\frac{1}{2}}\alpha^{-\frac{1}{2}}\right) + (1-\alpha)\,\frac{1}{2}c^{\frac{1}{2}}\alpha^{-\frac{3}{2}} = 0$$

[1] It suffices to derive an equality that implicitly defines α, as long as the equality contains no other endogenous variable. That is, solving for a closed-form expression for the optimal α is not required.

or

$$2\alpha^{\frac{3}{2}}\left(1-c^{\frac{1}{2}}\alpha^{-\frac{1}{2}}\right)=(1-\alpha)\,c^{\frac{1}{2}}$$

or

$$2\alpha^{\frac{3}{2}}-2c^{\frac{1}{2}}\alpha=c^{\frac{1}{2}}-c^{\frac{1}{2}}\alpha$$

or

$$2\alpha^{\frac{3}{2}}-c^{\frac{1}{2}}\alpha-c^{\frac{1}{2}}=0.$$

Part (b)

(b) What is the condition that we need to impose on the model to ensure that the principal's optimal contract is such that the agent's payment is strictly increasing in the level of output that is realized? For what values of x_A and x_B is this condition satisfied? Explain the intuition for why this condition matters.

- The condition is called the monotone likelihood ratio property (MLRP). It requires that the so-called likelihood ratio—defined as $\frac{\pi_{i1}}{\pi_{i0}}$—is increasing in the output level.
  - The book uses a slightly different definition ($\frac{\pi_{i1}-\pi_{i0}}{\pi_{i1}}$), and that would also be a valid answer.
  - In the course we have not been very careful about if/when we should have weak or strict inequalities, and this issue is not important for a valid answer.
- For MLRP to be satisfied in this example, the following four inequalities must be satisfied:

$$\frac{\pi_{21}}{\pi_{20}}>\frac{\pi_{11}}{\pi_{10}}\quad\text{and}\quad\frac{\pi_{31}}{\pi_{30}}>\frac{\pi_{21}}{\pi_{20}}\quad\text{and}\quad\frac{\pi_{41}}{\pi_{40}}>\frac{\pi_{31}}{\pi_{30}}\quad\text{and}\quad\frac{\pi_{51}}{\pi_{50}}>\frac{\pi_{41}}{\pi_{40}}.$$

  The first inequality is satisfied if

$$\frac{\pi_{21}}{\pi_{20}}>\frac{\pi_{11}}{\pi_{10}}\Leftrightarrow\frac{0.2-x_A}{0.2}>\frac{0.2-2x_B}{0.2}\Leftrightarrow x_A<2x_B.$$

  The second inequality is satisfied if

$$\frac{\pi_{31}}{\pi_{30}}>\frac{\pi_{21}}{\pi_{20}}\Leftrightarrow\frac{0.2+x_A}{0.2}>\frac{0.2-x_A}{0.2}\Leftrightarrow x_A>0.$$

  The third inequality is satisfied if

$$\frac{\pi_{41}}{\pi_{40}}>\frac{\pi_{31}}{\pi_{30}}\Leftrightarrow\frac{0.3+x_B}{0.3}>\frac{0.2+x_A}{0.2}\Leftrightarrow x_A<\frac{2}{3}x_B.$$

  The fourth inequality is satisfied if

$$\frac{\pi_{51}}{\pi_{50}}>\frac{\pi_{41}}{\pi_{40}}\Leftrightarrow\frac{0.1+x_B}{0.1}>\frac{0.3+x_B}{0.3}\Leftrightarrow x_B>0.$$

Summing up: the question constrains x_A and x_B to be in the interval $[0, 0.1]$; in addition we must impose the constraints $x_A > 0$, $x_B > 0$, and $x_A < \frac{2}{3}x_B$ for MLRP to be satisfied.

- Again, in the course we have not been very careful about if/when we should have weak or strict inequalities, and this issue is not important for a valid answer. However, the student should be consistent within her or his own answer.

- The intuition for why MLRP matters can be understood as follows. When choosing which output level to reward with a relatively large payment, the principal will consider the effect this choice has on the agent's incentive to choose the high effort level. In order to make that incentive as strong as possible, the principal should reward an output level for which choosing a high effort has a big (positive) impact on the probability of obtaining that output level. To see this, suppose $x_B = 0$ and $x_A = 0.1$ in the above example. Then the highest output level (y_5) would realize with the same probability (namely, 0.1), regardless of whether the agent exerts a low or a high effort. Therefore, rewarding the highest output level with a large payment would certainly not create an incentive for the agent to choose the high effort. However, if the principal awarded the third output level (y_3) with a large payment, then the agent would know he could increase the likelihood of receiving that payment with a positive amount (from 0.2 to 0.3) by choosing to make a high effort, and his incentive to do so would therefore be strong. In other words, the principal will have an incentive to reward an outcome for which π_{i1} (the high-effort probability) is large relative to π_{i0} (the low-effort probability)—formal analysis shows that it's the ratio that matters. As the MLRP condition guarantees that this ratio in larger for high output levels than for lower ones, it ensures that the payments also are higher (or at least not lower) for higher output levels.